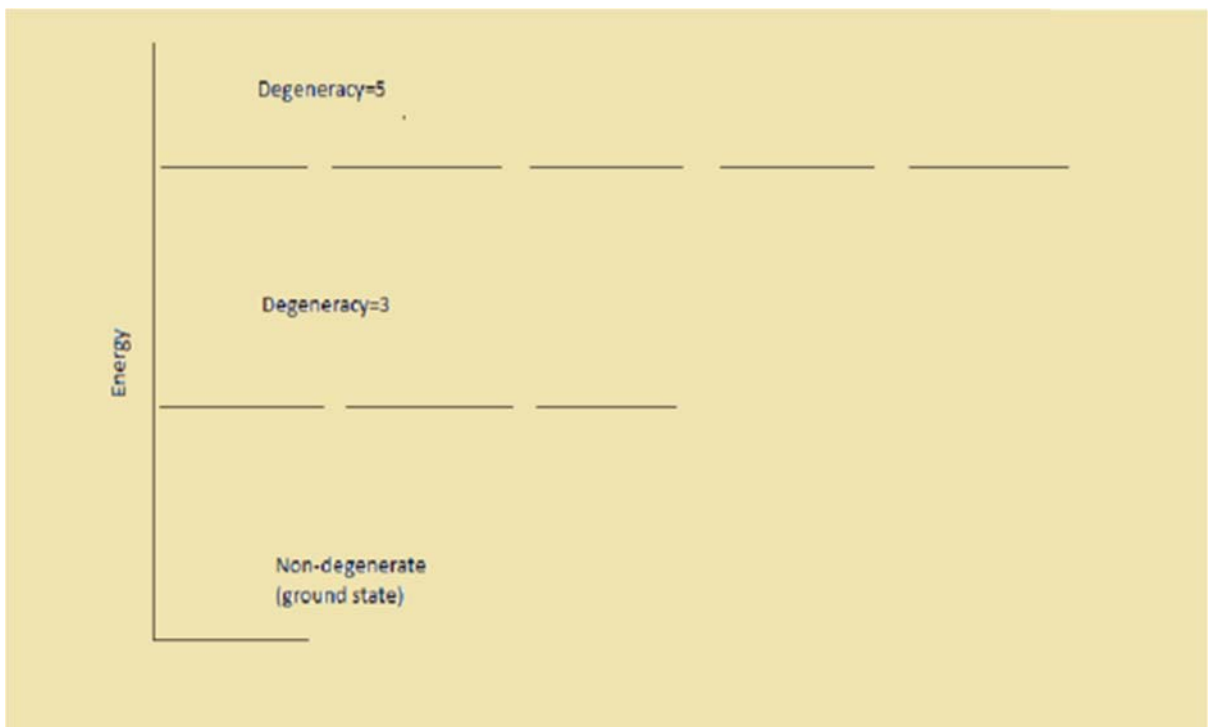


## Degeneracy of Hydrogen atom

In quantum mechanics, an energy level is said to be degenerate if it corresponds to two or more different measurable states of a quantum system. Conversely, two or more different states of a quantum mechanical system are said to be **degenerate** if they give the same value of energy upon measurement.

The number of different states corresponding to a particular energy level is known as the degree of degeneracy of the level. It is represented mathematically by the Hamiltonian for the system having more than one linearly independent eigenstate with the same eigenvalue.

In classical mechanics, this can be understood in terms of different possible trajectories corresponding to the same energy. Take for instance, in projectile motion, we can project the particle in different trajectories each having the same total energy as the particle travel through its parabolic path.



Picture extracted from Wikipedia “Degenerate Energy Level”

Thus for the degree of degeneracy of Hydrogen atom, we need to calculate how many possible linearly independent energy eigenstates corresponds to energy  $E_n$  of the atom. We knew that for hydrogen atom, its energy eigenstates is given by  $\Psi_{nlm}$  where:

Principal quantum number  $n = 1, 2, 3, \dots$

Azimuthal or Orbital Angular momentum quantum number  $l = 0, 1, 2, \dots, n-1$  i.e.  $n$  choices of them

Magnetic Quantum number  $m = 0, \pm 1, \pm 2, \dots, \pm l$  i.e.  $2l + 1$  choices of them.

Hence for a given  $n$ ,

Total number of possible energy eigenstates ( $nlm$ ) =

$$\sum_{l=0}^{n-1} (2l + 1) = 1 + \sum_{l=1}^{n-1} (2l + 1) = 1 + 2 \sum_{l=1}^{n-1} l + \sum_{l=1}^{n-1} 1 = 1 + 2 \left( \frac{(n-1)n}{2} \right) + (n-1) = n^2$$

Therefore

The degree of degeneracy of Hydrogen atom =  $n^2$

We know that electrons have two different type of spins (spin up and spin down). Putting that into our degeneracy result, we have

The  $n$ th shell of Hydrogen atom can hold up to  $2n^2$  electrons

The first shell (K) can hold up to two electrons.

The second shell (L) can hold up to eight 8 electrons.

The third shell (M) can hold up to 18 electrons

The fourth shell (N) can hold up to 32 electrons and so on.