2015 PH1012 Tut 1: Basics / Thermal Physics / Archimedes' Principle

1. Giancoli P13.7 How The Earth is not a uniform sphere, but has regions of varying density. Consider a simple model of the Earth divided into three regions—inner core, outer core, and mantle. Each region is taken to have a unique constant density (the average density of that region in the real Earth):

<table>
<thead>
<tr>
<th>Region</th>
<th>Radius (km)</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Core</td>
<td>0–1220</td>
<td>13,000</td>
</tr>
<tr>
<td>Outer Core</td>
<td>1220–3480</td>
<td>11,100</td>
</tr>
<tr>
<td>Mantle</td>
<td>3480–6371</td>
<td>4,400</td>
</tr>
</tbody>
</table>

(a) Use this model to predict the average density of the entire Earth.
(b) The measured radius of the Earth is 6371 km and its mass is $5.98 \times 10^{24}$ kg. Use these data to determine the actual average density of the Earth and compare it (as a percent difference) with the one you determined in (a).

2. Knight(2) P15.5 The deepest point in the ocean is 11 km (deeper than Mount Everest is tall). What is the pressure in atmosphere at this depth? [Please check out the values of relevant quantities needed by yourself].

3. Knight(2) P15.9 A research submarine has a 20-cm-diameter window 8.0 cm thick. The manufacturer says that the window can withstand forces up to $1.0 \times 10^6$ N. What is the submarine's maximum safe depth? The pressure inside the submarine is maintained at 1.0 atm.

4. Young and Freedman P12.16 The liquid in the open-tube manometer in the figure is mercury (density = 13 600 kg m⁻³). The lengths $y_1 = 3.00$ cm and $y_2 = 7.00$ cm. Atmospheric pressure is 980 millibars.

(a) What is the absolute pressure at the bottom of the U-shaped tube?
(b) What is the absolute pressure in the open tube at a depth of 4.00 cm below the free surface?
(c) What is the absolute pressure of the gas at the container?
(d) What is the gauge pressure of the gas in Pascals?
Knight(2) P15.41 The 70 kg student in the figure balances a 1200 kg elephant on a hydraulic lift with 2m diameter. What is the diameter of the piston the student is standing on?

Giancoli P13.39 (modified) How many helium-filled balloons would it take to lift a person? Assume the person has a mass of 75 kg and that each helium-filled balloon is spherical with a diameter of 33 cm. (Discussion) Do you need more or less helium balloons on the moon for the same purpose?

Giancoli P13.35 (modified) The specific gravity of pure ice is 0.917, whereas that of seawater is 1.025. What percent of an iceberg is above the surface of the water? (Discussion) If some ice and seawater are placed in a measuring cylinder, will the seawater level rise, drop or stay the same when all the ice melts?

[Activity] When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?

[Activity] A spherical iron ball bearing of diameter 4 mm is placed on a light piece of Styrofoam floating in a cylinder of oil as shown in the figure below. You may assume that the weight of the Styrofoam is negligible. Density of iron is 7.86 g cm\(^{-3}\); density of oil is 0.83 g cm\(^{-3}\).

(a) Determine the maximum load that the Styrofoam cup can carry before it sinks.
(b) Calculate the upthrust acting on the piece of Styrofoam.
(c) What is the volume of oil displaced by the piece of Styrofoam and the iron ball bearing?
(d) The ball bearing rolls off the piece of Styrofoam and falls into the oil. Will the oil level be higher, lower or the same as before? Explain your answer.

Answers:
1. (a) 5510 kg/m\(^3\); (b) 5520 kg/m\(^3\), 0.33.
2. 1100 atmospheres! 3.2 km
4. 1.07 \times 10^5 \text{ Pa}; 1.03 \times 10^5 \text{ Pa}; 1.03 \times 10^5 \text{ Pa}; 5.33 \times 10^3 \text{ Pa};
5. 0.48 m 3600 10.5% 8. -.
9. (b) 2.58 \times 10^{-3} N  (c) 0.317 \text{ cm}^3  (d) Lower

2
Q1. \( m_1 = \rho_1 V_1 = \rho_1 \frac{4}{3} \pi r_1^3 \). 
\( m_2 = \rho_2 V_2 = \rho_2 \left[ \frac{4}{3} \pi r_2^3 - \frac{4}{3} \pi r_1^3 \right] \)
\( m_2 = \frac{4}{3} \pi \rho_2 \left( r_2^3 - r_1^3 \right) \)
\( m_3 = \rho_3 V_3 = \rho_3 \left[ \frac{4}{3} \pi r_3^3 - \frac{4}{3} \pi r_2^3 \right] \)
\( m_3 = \frac{4}{3} \pi \rho_3 \left( r_3^3 - r_2^3 \right) \)

Total mass = \( m_1 + m_2 + m_3 \)

\[
M = \frac{4}{3} \pi \rho_1 r_1^3 + \frac{4}{3} \pi \rho_2 \left( r_2^3 - r_1^3 \right) + \frac{4}{3} \pi \rho_3 \left( r_3^3 - r_2^3 \right)
\]

\[
= \frac{4}{3} \pi \left[ \left( \rho_1 - \rho_2 \right) r_1^3 + \left( \rho_2 - \rho_3 \right) r_2^3 + \rho_3 r_3^3 \right]
\]

\[
= \frac{4}{3} \pi \left[ \left( 13,000 - 11,100 \right) (1220 \times 10^3)^3 + \left( 11,100 - 4,200 \right) (3480 \times 10^3)^3 \right.
\]

\[
+ \left. 4,200 \times (6371 \times 10^3)^3 \right] \times 10^9 \text{ kg}
\]

\approx 5.96333 \times 10^{24} \text{ kg}

Total Volume = \( \frac{4}{3} \pi r_3^3 \)

Average Density = \( \frac{\text{Total Mass}}{\text{Total Vol}} = \frac{5.96333 \times 10^{24}}{\frac{4}{3} \pi (6371 \times 10^3)^3} \)

\approx 5505.258903 \text{ kg m}^{-3}

\approx 5510 \text{ kg m}^{-3}

1.1
(b2. Actual average density = \frac{\text{Total Mass}}{\text{Total Vol.}}
\[= \frac{5.98 \times 10^{24} \text{ kg}}{\frac{4}{3} \pi (6371 \times 10^3)^3} = 5520 - 644.216}\]
\[= 5520 \text{ kg/m}^3\]

\% \text{ difference} = \frac{5520 - 5510}{5520} \times 100\% = 0.18\%
\text{OR: } \% \text{ difference} = \frac{5520 - 644.216 - 5505 - 258903}{5520 - 644.216} \times 100\%
\[= 0.28\%
\text{OR: } \% \text{ difference} = 0.3\%\]

2. \[P = P_0 + h \rho g\]
\[= P_0 + \frac{11030 \times 1030 \times 9.81}{1.013 \times 10^5} P_0\]
\[= 1098 P_0\]
\[= 1100 P_0\]

(P_0 = \text{atm } = 1.013 \times 10^5 \text{ Pa})

3. \[P = \frac{\rho A}{h} = h \rho g\]
\[\frac{10^6}{\pi \times 0.12} = h \times 1030 \times 9.81\]
\[\Rightarrow h \approx 3.150 \text{ m } \approx 3.2 \text{ km}\]
\[ P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \]

\[ \frac{70 \times 9.81}{\pi r_1^2} = \frac{120 \times 9.81}{\pi \times 1^2} \]

\[ r_1^2 = \frac{70}{120} \Rightarrow r_1 = 0.34 \text{ m} \Rightarrow D = 0.48 \text{ m} \]

\[ DL = \alpha L_0 \Delta T \]

\[ = (0.20 \times 10^{-6}) (1.6) \Delta T \]

\[ = 1.6 \times 10^{-6} \text{ m} \]

\[ \frac{\text{Ratio}}{1.6 \times 10^{-6}} = 60. \]

\[ \alpha : \text{Coefficient of linear expansion} \]

\[ V = V_0 (1 + \beta \Delta T) \]

\[ = 1 (1 + 950 \times 10^{-6} \times 35) \]

\[ = 1.03328 \text{ m}^3. \]

\[ \rho = \frac{0.68 \times 10^3}{1.03325} = 658 \text{ kg m}^{-3} \]

\[ \approx 660 \text{ kg m}^{-3} \]

\[ \% \Delta \rho = \frac{660 - 660}{660} \times 100 \% = -3 \% \]

Consider man & balloons as an item. Forces acting on man & balloons.

\[ m_1 g + m_2 g = U_1 + U_2 \]

\[ \Rightarrow (M_1 + m_2) g = U_2 \]

\[ 75 + N (V_2) \Phi_H g = N V_2 \Phi_A g \]

\[ 75 = m_1 g = N V_2 g (\Phi_A - \Phi_H) \]

\[ N = \frac{m_1}{(\frac{4}{3} \pi r^3) (\Phi_A - \Phi_H)} \]

\[ \Phi_H = 0.179 \text{ Kg m}^{-3} \]

\[ \Phi_A = 1.29 \text{ Kg m}^{-3} \]

\[ N = \frac{3600}{(\frac{4}{3} \pi r^3) (\Phi_A - \Phi_H)} = 3837 \]
6. Discussion: Not possible as there is little arm on moon, thus the upthrust is negligible on moon.

7. Specific Gravity (SG) = \frac{\text{Substance Density}}{\text{Water Density}}

SG ice = 0.917  \quad \text{SG seawater} = 1.025

\[
\frac{V_f}{V_i} = \frac{\rho_f}{\rho_i} = \frac{\frac{\text{SG}_i \cdot g \cdot w}{\text{SG}_f \cdot g \cdot w}}{\frac{\text{SG}_i \cdot g \cdot w}{\text{SG}_f \cdot g \cdot w}} = \frac{0.917}{1.025} = 0.895
\]

\[
\Rightarrow \% \text{ submerged} = 89.5\%
\]
\[
\Rightarrow \% \text{ above seawater} = 100 - 89.5 = 10.5\%
\]

**Discussion:**

\[
U = m_u g
\]
\[
m_f g = m_u g
\]
\[
\rho_f V_f = \rho_i V_i
\]
\[
V_f = \frac{\rho_i}{\rho_f} V_i
\]

Since \( \rho_i < \rho_f \) \(\Rightarrow V_u > V_f \), \(4 \Rightarrow \text{seawater level rises}\)
\[ T + U = W. \]

\[ m_1g + V_0 \rho_i g_2 = m_2g \]

\[ V_f = \frac{m_2g - m_1g_i}{\rho_i g_2} = \frac{m - m_i}{\rho_f} \]

\[ V_f = \frac{14.7 - 13.4}{1 / \text{sec}} \]

\[ = 6.3 \times 10^{-3} \text{ m}^3 \]

Density of crown = \[ \frac{14.7}{1.3 \times 10^{-3}} \]

\[ = 11308 \text{ kg/m}^3 \]

\[ = 1.13 \times 10^4 \text{ kg/m}^3 \]

\[ = 19.3 \times 10^{-3} \text{ kg/m}^3 \]

\[ \Rightarrow \] Crown not made of gold.
(b) \( U = mg = \left( \frac{2}{3} \pi r^3 \right) \rho g \)

\[
= \frac{4}{3} \pi \left( 2 \times 10^{-3} \right)^3 \left( 9.81 \right) \times \frac{7.86 \times 10^{-3}}{(10^{-2})^2} \\
= 2.58 \times 10^{-3} \text{ N}
\]

(c) **Archimedes Principle:**

\( \text{Upthrust} = \text{wt of fluid displaced} \)

\[
2.58 \times 10^{-3} = \int V \rho g \\
= \frac{0.83 \times 10^{-3}}{(10^{-2})^2} V \times 9.81 \\
V = 3.17 \times 10^{-7} \text{ m}^3 \\
= 0.317 \text{ cm}^3
\]

(d) \( \text{Bar will into cylinder:} \)

\( \text{Vol of oil displaced} = \frac{4}{3} \pi r^3 \)

\[
= \frac{4}{3} \pi \left( 2 \times 10^{-3} \right)^3 = 3.35 \times 10^{-8} \text{ m}^3 < 3.19 \times 10^{-7} \text{ m}^3
\]

\( \Rightarrow \) less oil is displaced

\( \Rightarrow \) oil level is lower.
Floating: \( U_1 = mg \)

Sinking: \( R + U_2 = mg \) \( \Rightarrow U_2 = mg - R \)

\( U_2 < U_1 \)

\( \Rightarrow \) less oil is displaced in sink position.

\( \Rightarrow \) oil level lower in sink position.

Note: When sinking,

\[ U_2 + mg = \]

\[ mg - U_2 = ma. \]
4. (a) \[ P = P_{\text{atm}} + \frac{1}{2} \rho g \]
\[ = 9.8 \times 10^4 + (7 \times 10^{-2}) (13600) (9.8) \]
\[ = 1.07 \times 10^5 \text{ Pa} \]

(b) \[ P = P_{\text{atm}} + (4 \times 10^{-2}) (13600) (9.8) \]
\[ = 1.03 \times 10^5 \text{ Pa} \]

(c) \[ P + \frac{1}{2} \rho g = P_{\text{atm}} + \frac{1}{2} \rho g \]
\[ P = 1.07 \times 10^5 - (3 \times 10^{-2}) (13600) (9.8) \]
\[ = 1.03 \times 10^5 \text{ Pa} \]

(d) Gauge Pressure = \((y_2 - y_1) \rho g\)
\[ = \left[ (7 - 3) \times 10^{-2} \right] (13600) (9.8) \]
\[ = 5331.2 \text{ Pa} \]
\[ = 5.33 \times 10^3 \text{ Pa} \]